Multistationarity in Structured Reaction Networks

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Many dynamical systems arising in applications exhibit multistationarity (two or more positive steady states), but it is often difficult to determine whether a given system is multistationary, and if so to identify a witness to multistationarity, that is, specific parameter values for which the system exhibits multiple steady states. In this talk, we introduce a procedure to investigate multistationarity and to find a witness. In practice, the procedure is much less expensive than traditional quantifier elimination. Our method is based on two new sufficient conditions for multistationarity. First, when there are no boundary steady states and a positive steady-state parametrization exists, one can conclude multistationarity if a certain critical function changes sign. Particularly, if the steady states are defined by binomials, we have multistationarity if a certain critical function contains terms with different signs. Second, when the steady-state equations can be replaced by equivalent triangular-form equations, we have multistationarity if a positive degenerate steady state exists. We also investigate the mathematical structure of this critical function, and give conditions that guarantee that triangular-form equations exist by studying the specialization of Grobner bases.